

Mass spectra and decay properties of D_s Meson in a relativistic Dirac formalism

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The mass spectra of D_s meson is obtained in the framework of relativistic independent quark model using Martin like potential for the quark confinement. The predicted excited states are in good agreement with the experimental results as well as with the lattice and other theoretical predictions. The spectroscopic parameters are employed further to compute the decay constant, electromagnetic transition and leptonic decay widths. The present result for its decay constant, f_P (252.82 MeV) is in excellent agreement with the value 252.6 ± 11.1 MeV reported by CLEO-c and the predicted branching ratios for ($D_s \rightarrow \tau \bar{\nu}_\tau, \mu \bar{\nu}_\mu$) ($5.706 \times 10^{-2}, 5.812 \times 10^{-3}$) are in close agreement with the PDG values ($(5.43 \pm 0.31) \times 10^{-2}, (5.90 \pm 0.33) \times 10^{-3}$) respectively.

I. INTRODUCTION

Having played a major role in the foundation of QCD, heavy hadron spectroscopy has witnessed in the last few years a renewal of interest due to many new states observed in recent years. The remarkable progress at the experimental [1] side, with various high energy machines such as BaBar, BELLE, BES-III, B-factories, Tevatron, ARGUS collaborations, CLEO, CDF, SELEX, DØ etc., for the study of hadrons has opened up new challenges in the theoretical understanding of light-heavy flavour hadrons. Study of D_s meson carry special interest as it is a hadron with two open flavours (c, \bar{s}) that restricts its decay via strong interactions. These particles thus provide us a clean laboratory to study electromagnetic and weak interaction. The discoveries of new resonances of D_s states such as $D_s(2638)$ [2], $D_s(2710)$ [3], $D_s(2860)$ [4], $D_s(3040)$ [4] etc., have further generated considerable interest towards the spectroscopy of this double open flavour meson. The masses of low-lying $1S$ and $1P_J$ states of D_s mesons are recorded both experimentally [1] and theoretically [5–13]. However, the existing results on excited heavy-light mesons are partially inconclusive and even contradictory in several cases.

Thus any attempts towards the understanding of these newly observed states become very important for better understanding the quark-antiquark dynamics within $Q\bar{q}$ bound state. So, a successful theoretical model can provide important information about the quark-antiquark interactions and the behavior of QCD within the doubly open flavour at the hadronic system. Though there exist many theoretical models to study the hadron properties based on its quark structure, the predictions for low-lying states are off by 60 – 90 MeV with respect to the respective experimental values. Moreover the issue related to the hyperfine and fine structure splitting of the mesonic states; their intricate dependence with the constituent quark masses and the

running strong coupling constant are still unresolved. Though the validity of nonrelativistic models is very well established and significantly successful for the description of heavy quarkonia, it seemed to fail for the description of meson containing light flavour quarks or antiquarks.

Apart from the successful predictions of the mass spectra, validity of any phenomenological model depends also on the successful predictions of their decay properties. For better predictions of the decay widths, many models have incorporated additional contributions such as radiative and higher order QCD corrections [12, 14–17]. Thus, in this paper we make an attempt to study properties like mass spectrum, decay constants and other decay properties of the D_s meson based on a relativistic Dirac formalism. We investigate the heavy-light mass spectra of $D_s(c\bar{s})$ meson in this framework with Martin like confinement potential.

Along with the mass spectra, the pseudoscalar decay constants of the heavy-light mesons have also been estimated in the context of many QCD-motivated approximations. The predictions of such methods cover a wide range of values [18, 19]. It is important to have reliable estimate of the decay constant as it is an important parameter in many weak processes such as quark mixing, CP violation, etc. The leptonic decay of charged meson is another important annihilation channel through the exchange of virtual W^\pm boson. Though this annihilation process is rare, but they have clear experimental signatures due to the presence of highly energetic leptons in the final state. And there exist experimental observations of the leptonic decays of D_s meson. The leptonic decays of mesons entails an appropriate representation of the initial state of the decaying vector mesons in terms of the constituent quark and antiquark with their respective momenta and spin. The bound constituent quark and antiquark inside the meson are in definite energy states having no definite momenta. However one can find out the momentum distribution amplitude for the constituent quark and antiquark inside the meson immediately before their annihilation to a lepton pair. Thus, it is appropriate to compute the leptonic branching ratio

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here and compare our result with the experimental values as well as with the predictions based on other models.

II. THEORETICAL FRAMEWORK

The quark confining interaction of meson is considered to be produced by the non-perturbative multigluon mechanism and this mechanism is unfeasible to estimate theoretically from first principles of QCD. In the present study, we assume that the constituent quarks inside a meson is independently confined by an average potential of the form [20]

$$V(r) = \frac{1}{2}(1 + \gamma_0)(\lambda r^{0.1} + V_0) \quad (1)$$

To first approximation, the confining part of the interaction is believed to provide the zeroth-order quark dynamics inside the meson through the quark Lagrangian density

$$\mathcal{L}_q^0(x) = \bar{\psi}_q(x) \left[\frac{i}{2} \gamma^\mu \vec{\partial}_\mu - V(r) - m_q \right] \psi_q(x). \quad (2)$$

In the stationary case, the spatial part of the quark wave functions $\psi(\vec{r})$ satisfies the Dirac equation given by

$$[\gamma^0 E_q - \vec{\gamma} \cdot \vec{P} - m_q - V(r)] \psi_q(\vec{r}) = 0. \quad (3)$$

The solution of Dirac equation can be written as two component (positive and negative energies in the zeroth order) form as

$$\psi_{nlj}(r) = \begin{pmatrix} \psi_{nlj}^{(+)} \\ \psi_{nlj}^{(-)} \end{pmatrix} \quad (4)$$

where

$$\psi_{nlj}^{(+)}(\vec{r}) = N_{nlj} \begin{pmatrix} ig(r)/r \\ (\sigma \cdot \hat{r})f(r)/r \end{pmatrix} \mathcal{Y}_{ljm}(\hat{r}) \quad (5)$$

$$\psi_{nlj}^{(-)}(\vec{r}) = N_{nlj} \begin{pmatrix} i(\sigma \cdot \hat{r})f(r)/r \\ g(r)/r \end{pmatrix} (-1)^{j+m_j-l} \mathcal{Y}_{ljm}(\hat{r}) \quad (6)$$

and N_{nlj} is the overall normalization constant. The normalized spin angular part is expressed as

$$\mathcal{Y}_{ljm}(\hat{r}) = \sum_{m_l, m_s} \langle l, m_l, \frac{1}{2}, m_s | j, m_j \rangle Y_l^{m_l} \chi_{\frac{1}{2}}^{m_s} \quad (7)$$

Here the spinor $\chi_{\frac{1}{2} m_s}$ are eigenfunctions of the spin operators,

$$\chi_{\frac{1}{2} \frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_{\frac{1}{2} -\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (8)$$

The reduced radial part $g(r)$ of the upper component and $f(r)$ of the lower component of Dirac spinor $\psi_{nlj}(r)$ are

the solutions of the equations given by

$$\frac{d^2 g(r)}{dr^2} + \left[(E_D + m_q)[E_D - m_q - V(r)] - \frac{\kappa(\kappa + 1)}{r^2} \right] g(r) = 0 \quad (9)$$

and

$$\frac{d^2 f(r)}{dr^2} + \left[(E_D + m_q)[E_D - m_q - V(r)] - \frac{\kappa(\kappa - 1)}{r^2} \right] f(r) = 0 \quad (10)$$

It can be transformed in to a convenient dimensionless form given as [21]

$$\frac{d^2 g(\rho)}{d\rho^2} + \left[\epsilon - \rho^{0.1} - \frac{\kappa(\kappa + 1)}{\rho^2} \right] g(\rho) = 0 \quad (11)$$

and

$$\frac{d^2 f(\rho)}{d\rho^2} + \left[\epsilon - \rho^{0.1} - \frac{\kappa(\kappa - 1)}{\rho^2} \right] f(\rho) = 0 \quad (12)$$

In terms of dimensionless variable $\rho = (r/r_0)$ with the arbitrary scale factor chosen conveniently as

$$r_0 = \left[(m_q + E_D) \frac{\lambda}{2} \right]^{-\frac{10}{21}}, \quad (13)$$

and a corresponding dimensionless energy eigenvalue defined as

$$\epsilon = (E_D - m_q - V_0)(m_q + E_D)^{\frac{1}{21}} \left(\frac{2}{\lambda} \right)^{\frac{20}{21}} \quad (14)$$

Here, it is suitable to define a quantum number κ by

$$\kappa = \begin{cases} -(\ell + 1) & = -(j + \frac{1}{2}) & \text{for } j = \ell + \frac{1}{2} \\ \ell & = +(j + \frac{1}{2}) & \text{for } j = \ell - \frac{1}{2} \end{cases} \quad (15)$$

Equations (11) and (12) now can be treated similar to radial Schrödinger equation with a potential ρ^ν which can be solved numerically [22].

The solutions $g(\rho)$ and $f(\rho)$ are normalized to get

$$\int_0^\infty (f_q^2(r) + g_q^2(r)) dr = 1. \quad (16)$$

The wavefunction for a $D_s(c\bar{s})$ meson now can be constructed using Eqn (5) and (6) and the corresponding mass of the quark-antiquark system can be written as

$$M_{Q\bar{q}} = E_D^Q + E_D^{\bar{q}} \quad (17)$$

where $E_D^{Q/\bar{q}}$ are obtained using Eqn. (14) and (15). For the spin triplet (vector) and spin singlet (pseudoscalar) state, the choices of (j_1, j_2) are $((l_1 + \frac{1}{2}), (l_1 + \frac{1}{2}))$ and $((l_1 + \frac{1}{2}), (l_1 - \frac{1}{2}))$ respectively. The previous work of independent quark model within the Dirac formalism by [20] has been extended here by incorporating the spin-spin, spin-orbit and tensor interactions of the confined one gluon exchange potential (COGEP) [23, 24], in addition to the j-j coupling of the quark-antiquark. Finally,

TABLE I. The fitted model parameters for the D_s systems

System Parameters	D_s
Quark mass (in GeV)	$m_s = 0.1$ and $m_c = 1.27$
Potential parameter(λ)	$2.2655 + B \text{ GeV}^{\nu+1}$
V_0	- 2.6155 GeV
Centrifugal parameter (B)	$(n * 0.153) \text{ GeV}^{-1}$ for $l = 0$ $((n + l) * 0.1267) \text{ GeV}^{-1}$ for $l \neq 0$
σ ($j - j$ coupling constant)	0.0055 GeV^3 for $l = 0$ 0.2696 GeV^3 for $l \neq 0$

the mass of the specific $^{2S+1}L_J$ states of $Q\bar{q}$ system is expressed as

$$M_{2S+1L_J} = M_{Q\bar{q}} (n_1 l_1 j_1, n_2 l_2 j_2) + \langle V_{Q\bar{q}}^{j_1 j_2} \rangle + \langle V_{Q\bar{q}}^{LS} \rangle + \langle V_{Q\bar{q}}^T \rangle \quad (18)$$

Here, the spin-spin part is defined as

$$\langle V_{Q\bar{q}}^{j_1 j_2}(r) \rangle = \frac{\sigma \langle j_1 j_2 JM | \hat{j}_1 \cdot \hat{j}_2 | j_1 j_2 JM \rangle}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \quad (19)$$

where σ is the $j - j$ coupling constant. The expectation value of $\langle j_1 j_2 JM | \hat{j}_1 \cdot \hat{j}_2 | j_1 j_2 JM \rangle$ contains the $(j_1 j_2)$ coupling and the square of Clebsch-Gordan coefficients. The tensor and spin-orbit parts of confined one-gluon exchange potential (COGEP) [23, 24] is given by

$$V_{Q\bar{q}}^T(r) = -\frac{\alpha_s}{4} \frac{N_Q^2 N_{\bar{q}}^2}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \otimes \lambda_Q \cdot \lambda_{\bar{q}} \left(\left(\frac{D_1''(r)}{3} - \frac{D_1'(r)}{3r} \right) S_{Q\bar{q}} \right) \quad (20)$$

where $S_{Q\bar{q}} = [3(\sigma_Q \cdot \vec{r})(\sigma_{\bar{q}} \cdot \vec{r}) - \sigma_Q \cdot \sigma_{\bar{q}}]$ and $\vec{r} = \vec{r}_Q - \vec{r}_{\bar{q}}$ is the unit vector in the direction of \vec{r} and

$$V_{Q\bar{q}}^{LS}(r) = \frac{\alpha_s}{4} \frac{N_Q^2 N_{\bar{q}}^2}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \frac{\lambda_Q \cdot \lambda_{\bar{q}}}{2r} \otimes [[r \times (\hat{p}_Q - \hat{p}_{\bar{q}}) \cdot (\sigma_Q + \sigma_{\bar{q}})] (D_0'(r) + 2D_1'(r)) + [r \times (\hat{p}_Q + \hat{p}_{\bar{q}}) \cdot (\sigma_i - \sigma_j)] (D_0'(r) - D_1'(r))] \quad (21)$$

where α_s is the strong coupling constant and it is computed as

$$\alpha_s = \frac{4\pi}{(11 - \frac{2}{3} n_f) \log \left(\frac{E_Q^2}{\Lambda_{QCD}^2} \right)} \quad (22)$$

with $n_f = 3$ and $\Lambda_{QCD} = 0.150$ GeV. In Eqn. (21) the spin-orbit term has been split into symmetric ($\sigma_Q + \sigma_{\bar{q}}$) and anti-symmetric ($\sigma_Q - \sigma_{\bar{q}}$) spin-orbit terms.

We have adopted the same parametric form of the confined gluon propagators which are given by [23, 24]

$$D_0(r) = \left(\frac{\alpha_1}{r} + \alpha_2 \right) \exp(-r^2 c_0^2/2) \quad (23)$$

and

$$D_1(r) = \frac{\gamma}{r} \exp(-r^2 c_1^2/2) \quad (24)$$

with $\alpha_1 = 0.036$, $\alpha_2 = 0.056$, $c_0 = 0.1017$ GeV, $c_1 = 0.1522$ GeV, $\gamma = 0.0139$. Other optimized model parameters employed in the present study are listed in the Table I. The computed S-wave masses and other P-wave and D-wave masses of D_s meson states are listed in Table II and Table III respectively. Fig.(??) shows the energy level diagram of D_s meson spectra along with available experimental results.

III. MAGNETIC (M1) TRANSITIONS OF OPEN CHARM MESON

Spectroscopic studies led us to compute the decay widths of energetically allowed radiative transitions of the type $A \rightarrow B + \gamma$ among several vector and pseudoscalar states of D_s meson. The magnetic transition correspond to spin flip and hence the vector meson decay to pseudoscalar $V \rightarrow P\gamma$ represents a typical M1 transition. Such transitions are experimentally important to the identification of newly observed states. Assuming that such transitions are single vertex processes governed mainly by photon emission from independently confined quark and antiquark inside the meson, the S-matrix elements in the rest frame of the initial meson is written in the form

$$S_{BA} = \left\langle B\gamma \left| -ie \int d^4x T \left[\sum_q e_q \bar{\psi}_q(x) \gamma^\mu \psi_q(x) A_\mu(x) \right] \right| A \right\rangle. \quad (25)$$

The common choice of the photon field $A_\mu(x)$ is made here in Coulomb-gauge with $\epsilon(k, \lambda)$ as the polarization vector of the emitted photon having energy momentum $(k_0 = |\mathbf{k}|, \mathbf{k})$ in the rest frame of A. The quark field operators find a possible expansions in terms of the complete set of positive and negative energy solutions given by Eqs. (5) and (6) in the form

$$\Psi_q(x) = \sum_{\zeta} \left[b_{q\zeta} \psi_{q\zeta}^{(+)}(r) \exp(-iE_{q\zeta}t) + b_{q\zeta}^\dagger \psi_{q\zeta}^{(-)}(r) \exp(iE_{q\zeta}t) \right] \quad (26)$$

where the subscript q stands for the quark flavor and ζ represents the set of Dirac quantum numbers. Here $b_{q\zeta}$

TABLE II. S-wave D_s ($c\bar{s}$) spectrum (in MeV).

nL	J^P	State	$M_{Q\bar{q}}$	$\langle V_{Q\bar{q}}^{j_1 j_2} \rangle$	Present	Experiment					
						Meson	Mass[1]	[28]	[29]	[13]	[30]
1S	1^-	1^3S_1	2113.2	0.73	2113.9	D_s^*	2112.3 ± 0.5	. . .	2111	2117	2107
	0^-	1^1S_0	1970.1	-1.84	1968.3	D_s	1968.49 ± 0.32	. . .	1969	1970	1969
2S	1^-	2^3S_1	2717.3	0.46	2717.8	$D_s^*(2710)$	2710_{-7}^{+12} [31, 32]	2728	2731	2723	2714
	0^-	2^1S_0	2634.6	-1.06	2633.5	$D_s(2632)$	2632.5 ± 1.7 [2]	2656	2688	2684	2640
3S	1^-	3^3S_1	3263.5	0.33	3263.8		. . .	3200	3242	3180	. . .
	0^-	3^1S_0	3203.2	-0.75	3202.4		. . .	3140	3219	3158	. . .
4S	1^-	4^3S_1	3781.4	0.25	3781.6		3669	3571	. . .
	0^-	4^1S_0	3732.7	-0.57	3732.1		3652	3556	. . .

[28] - Semi-relativistic model
 [29] - Quasi potential Approach
 [13] - Relativistic quark-antiquark potential (Coulomb plus power) model
 [30] - Non-relativistic constituent quark model

and $b_{q\zeta}^\dagger$ are the quark annihilation and the antiquark creation operators corresponding to the eigenmodes ζ . After some standard calculations (the details of calculations can be found in Refs. [25, 26] and [27]), the S-matrix elements can be expressed as

$$S_{BA} = i\sqrt{\left(\frac{\alpha}{k}\right)} \delta(E_B + k - E_A) \sum_{q,m,m'} \langle B | \left[J_{m'm}^q(k, \lambda) b_{qm'}^\dagger b_{qm} - \tilde{J}_{mm'}^{\tilde{q}}(k, \lambda) \tilde{b}_{qm'}^\dagger \tilde{b}_{qm} \right] | A \rangle \quad (27)$$

Here $E_A = M_A$, $E_B = \sqrt{k^2 + M_B^2}$ and (m, m') are the possible spin quantum numbers of the confined quarks corresponding to the ground state of the mesons. We have

$$J_{m'm}^q(k, \lambda) = e_q \int d^3r \exp(-i\vec{k} \cdot \vec{r}) [\bar{\psi}_{qm'}(r) \vec{\gamma} \cdot \vec{\epsilon}(k, \lambda) \psi_{qm}(r)] \quad (28)$$

$$\tilde{J}_{mm'}^{\tilde{q}}(k, \lambda) = e_q \int d^3r \exp(-i\vec{k} \cdot \vec{r}) [\bar{\phi}_{qm}(r) \vec{\gamma} \cdot \vec{\epsilon}(k, \lambda) \phi_{qm'}(r)]. \quad (29)$$

One can reduce the above equations to simple forms as

$$J_{m'm}^q(k, \lambda) = -i \mu_q(k) [\chi_m^\dagger(\vec{\sigma} \cdot \vec{K}) \chi_m], \quad (30)$$

and

$$\tilde{J}_{mm'}^{\tilde{q}}(k, \lambda) = i \mu_q(k) [\tilde{\chi}_m^\dagger(\vec{\sigma} \cdot \vec{K}) \tilde{\chi}_m] \quad (31)$$

where $\vec{K} = \vec{k} \times \vec{\epsilon}(k, \lambda)$. Eqn. (27) further simplified to get

$$S_{BA} = i\sqrt{\left(\frac{\alpha}{k}\right)} \delta(E_B + k - E_A) \sum_{q,m,m'} \langle B | \mu_q(k) \left[\chi_{m'}^\dagger \vec{\sigma} \cdot \vec{K} \chi_m b_{qm'}^\dagger b_{qm} + \tilde{\chi}_m^\dagger \vec{\sigma} \cdot \vec{K} \tilde{\chi}_{m'} \tilde{b}_{qm'}^\dagger \tilde{b}_{qm} \right] | A \rangle \quad (32)$$

where $\mu_q(k)$ is expressed as

$$\mu_q(k) = \frac{2e_q}{k} \int_0^\infty j_1(kr) f_q(r) g_q(r) dr \quad (33)$$

where $j_1(kr)$ is the spherical Bessel function and the energy of the outgoing photon in the case of a vector meson undergoing a radiative transition to its pseudoscalar state, for instance, $D_s^* \rightarrow D_s \gamma$ is given by

$$k = \frac{M_{D_s^*}^2 - M_{D_s}^2}{2M_{D_s^*}} \quad (34)$$

The relevant transition magnetic moment is expressed as

$$\mu_{D_s^* D_s}(k) = \frac{1}{3} [2\mu_c(k) - \mu_s(k)], \quad (35)$$

Now, the Magnetic (M1) transition width of $D_s^* \rightarrow D_s \gamma$ can be obtained as

$$\Gamma_{D_s^* \rightarrow D_s \gamma} = \frac{4\alpha}{3} k^3 |\mu_{D_s^* D_s}(k)|^2 \quad (36)$$

The computed transition widths of low lying S-wave states are tabulated in Table VI and are compared with other model predictions.

TABLE III. P-wave and D-wave D_s ($c\bar{s}$) spectrum (in MeV).

nL	J^P	State	$M_{Q\bar{q}}$	$\langle V_{Q\bar{q}}^{j_1 j_2} \rangle$	$\langle V^T \rangle$	$\langle V^{LS} \rangle$	Present	Experiment					
								Meson	Mass [1]	[28]	[29]	[13]	[30]
1P	2^+	1^3P_2	2520.9	19.24	-3.71	48.23	2584.7	$D_{s2}(2573)$	2571.9 ± 0.8	. . .	2571	2566	2559
	1^+	1^3P_1	2520.9	25.65	18.54	-48.23	2516.9	$D_{s1}(2536)$	2535.12 ± 0.13	. . .	2536	2540	2510
	0^+	1^3P_0	2520.9	-38.47	-37.08	-96.46	2349.0	$D_{s0}(2317)$	2317.8 ± 0.6	. . .	2509	2444	2344
	1^+	1^1P_1	2421.7	13.84	0	0	2435.6	$D_{s1}(2460)$	2459.6 ± 0.6	. . .	2574	2530	2488
2P	2^+	2^3P_2	3018.3	13.87	-6.28	81.75	3107.6				3045	3142	3048
	1^+	2^3P_1	3018.3	18.50	31.40	-81.75	2986.4	$D_{sJ}(3040)$	3044^{+30}_{-9} [4]		3040	3067	3019
	0^+	2^3P_0	3018.3	-27.75	-62.8	-163.51	2764.3				2970	3054	2947
	1^+	2^1P_1	2949.4	9.64	0	0	2959.0				3020	3154	3023
3P	2^+	3^3P_2	3479.7	10.76	-8.53	111.06	3593.0			. . .	3580
	1^+	3^3P_1	3479.7	14.34	42.64	-111.06	3425.6			. . .	3519
	0^+	3^3P_0	3479.7	-21.51	-85.27	-222.13	3150.9			. . .	3513
	1^+	3^1P_1	3426.0	7.37	0	0	3433.4			. . .	3618
1D	3^-	1^3D_3	2952.7	-21.79	-0.03	0.49	2931.4	$D_{sJ}^*(2860)$	2862^{+6}_{-3} [4]	2840	2971	2834	2811
	2^-	1^3D_2	2952.7	-64.74	0.11	-0.25	2887.8			2885	2961	2816	2788
	1^-	1^3D_1	2952.7	-109.81	-0.11	-0.75	2842.0			2870	2913	2873	2804
	2^-	1^1D_2	2874.3	-2.65	0	0	2871.6			2828	2931	2896	2849
2D	3^-	2^3D_3	3423.7	-15.72	-0.03	0.52	3408.4			3285	3469	3263	3240
	2^-	2^3D_2	3423.7	-46.71	0.11	-0.26	3376.8			. . .	3456	3248	3217
	1^-	2^3D_1	3423.7	-79.23	-0.11	-0.79	3343.5			3290	3383	3292	3217
	2^-	2^1D_2	3363.7	-1.87	0	0	3361.8			. . .	3403	3312	3260
3D	3^-	3^3D_3	3870.9	-12.06	-0.04	0.60	3859.4		
	2^-	3^3D_2	3870.9	-35.85	0.13	-0.30	3834.9		
	1^-	3^3D_1	3870.9	-60.80	-0.13	-0.91	3809.1		
	2^-	3^1D_2	3821.7	-1.42	0	0	3820.3		

IV. DECAY CONSTANT OF D_s MESON

The decay constant of a meson is an important parameter in the study of leptonic or non-leptonic weak decay processes. The decay constant (f_p) of pseudoscalar state is obtained by parameterizing the matrix elements of weak current between the corresponding meson and the vacuum as [33]

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 c | P_\mu \rangle = i f_p P^\mu \quad (37)$$

It is possible to express the quark-antiquark eigenmodes in the ground state of the meson in terms of the corresponding momentum distribution amplitudes. Accordingly, eigenmodes, $\psi_A^{(+)}$ in the state of definite momentum p and spin projection s'_p can be expressed as

$$\psi_A^{(+)} = \sum_{s'_p} \int d^3 p G_q(p, s'_p) \sqrt{\frac{m}{E_p}} U_q(p, s'_p) \exp(i \vec{p} \cdot \vec{r}) \quad (38)$$

where $U_q(p, s'_p)$ is the usual free Dirac spinors.

In the relativistic quark model, the decay constant can be expressed through the meson wave function $G_q(p)$ in the momentum space [26, 34]

$$f_P = \left(\frac{3 |I_p|^2}{2 \pi^2 M_p J_p} \right)^{\frac{1}{2}} \quad (39)$$

Here M_p is mass of the pseudoscalar meson and I_p and J_p are defined as

$$I_p = \int_0^\infty dp p^2 A(p) [G_{q1}(p) G_{q2}^*(-p)]^{\frac{1}{2}} \quad (40)$$

TABLE IV. Comparison of Center of Mass in D_s meson in MeV.

M_{CW}	Present	[35]	[29]	Exp.
$\overline{1S}$	2077.5	$2045.4 \pm 0.215 \pm 0.293$	2075.5	2076.3
$\overline{2S}$	2696.7	...	2720.2	2690.6
$\overline{3S}$	3248.4	...	3236.2	...
$\overline{4S}$	3769.2	...	3664.7	...
$\overline{1^3P_J}$	2535.9	...	2552.4	2531.4
$\overline{1P}$	2510.8	...	2557.8	2513.4
$\overline{2^3P_J}$	3029.0	...	3107.2	...
$\overline{2P}$	3011.5	...	3118.9	...

TABLE V. Mass splitting in D_s meson in MeV.

Splitting	Present	[35]	[29]	Exp.
$1^3S_1 - 1^1S_0$	145.6	$133.1 \pm 1.0 \pm 1.9$	143	143.8 ± 0.4
$2^3S_1 - 2^1S_0$	84.3	$72 \pm 24 \pm 1$	43	...
$3^3S_1 - 3^1S_0$	61.4	...	23	...
$4^3S_1 - 4^1S_0$	49.5	...	17	...
$D_{s0}(2317) - \overline{1S}$	271.5	$341.2 \pm 7.7 \pm 4.8$	433.5	241.5 ± 0.8
$D_{s1}(2460) - \overline{1S}$	358.1	$459.8 \pm 6.4 \pm 6.4$	498.5	383.2 ± 0.8
$D_{s1}(2536) - \overline{1S}$	439.4	$494.6 \pm 9.2 \pm 6.9$	460.5	459.0 ± 0.5
$D_{s2}(2573) - \overline{1S}$	507.2	$536.7 \pm 9.2 \pm 7.5$	495.5	496.3 ± 1.0
$2^1S_0 - \overline{1S}$	556.0	$654.4 \pm 26.7 \pm 9.2$	612.5	...
$2^3S_1 - \overline{1S}$	640.3	$726.4 \pm 20.8 \pm 10.2$	655.5	632.7^{+9}_{-6}

$$J_p = \int_0^\infty dp p^2 [G_{q1}(p) G_{q2}^*(-p)] \quad (41)$$

respectively. Where,

$$A(p) = \frac{(E_{p1} + m_{q1})(E_{p2} + m_{q2}) - p^2}{[E_{p1} E_{p2} (E_{p1} + m_{q1})(E_{p2} + m_{q2})]^{\frac{1}{2}}} \quad (42)$$

and $E_{p_i} = \sqrt{k_i^2 + m_{q_i}^2}$.

The computed decay constants of D_s meson from $1S$ to $4S$ states are tabulated in Table VII. Present result for $1S$ state is compared with experimental as well as other model predictions. There are no model predictions available for comparison of the decay constants of the $2S$ to $4S$ states.

V. LEPTONIC DECAY OF THE OPEN HEAVY FLAVOUR MESONS

Charged mesons produced from a quark and anti-quark can decay to a charged lepton pair when these objects annihilate via a virtual W^\pm boson as given in Fig.(??). Though the leptonic decays of open flavour mesons belong to rare decay [36, 37], they have clear experimental

signatures due to the presence of highly energetic lepton in the final state. And such decays are very clean due to the absence of hadrons in the final state [38]. The leptonic width of D_s meson is computed using the relation given by

$$\Gamma(D_s^+ \rightarrow l^+ \nu_l) = \frac{G_F^2}{8\pi} f_{D_s}^2 |V_{cs}|^2 m_l^2 \left(1 - \frac{m_l^2}{M_{D_s}^2}\right)^2 M_{D_s} \quad (43)$$

in complete analogy to $\pi^+ \rightarrow l^+ \nu$. These transitions are helicity suppressed; i.e., the amplitude is proportional to m_l , the mass of the lepton l . The leptonic widths of D_s (1^1S_0 state) meson are obtained from Eqn.(43) where the predicted values of the pseudoscalar decay constant f_{D_s} along with the masses of M_{D_s} and the PDG value for $V_{cs} = 1.006$ are used. The leptonic widths for separate lepton channel are computed for the choices of $m_{l=\tau,\mu,e}$. The branching ratio of these leptonic widths are then obtained as

$$BR = \Gamma(D_s \rightarrow l^+ \nu_l) \times \tau \quad (44)$$

where τ is the experimental lifetime of the D_s meson. The respective leptonic widths are tabulated in Table VIII along with other model predictions as well as with the experimental values. Our results are found to be in accordance with the available experimental values.

TABLE VI. Magnetic (M1) transition of Open Charm Meson

Process	k (MeV)		Γ (keV)						
	Present	[13]	<i>Present</i>	PDG [1]	[13]	[39]	[40]	[41]	
(1S) $D_s^* \rightarrow D_s \gamma$	141.24	403	0.3443	< 4500	5.98	0.13	0.48	1.12	
(2S) $D_s^* \rightarrow D_s \gamma$	83.48	152	0.0134		0.35				
(3S) $D_s^* \rightarrow D_s \gamma$	61.21	91	0.0030		0.08				
(3S) $D_s^* \rightarrow D_s \gamma$	49.47	65	0.0010		0.03				

TABLE VII. Pseudoscalar decay constant (f_P) of D_s systems (in MeV).

	f_P			
	1S	2S	3S	4S
Present	252.81	336.56	391.74	433.16
PDG [1]	260.0 \pm 5.4			
Belle [42]	255.5 \pm 4.2 \pm 5.1			
BaBar [43]	258.6 \pm 6.4 \pm 7.5			
CLEO-c [44]	259.0 \pm 6.2 \pm 3.0			
CLEO-c [45]	252.6 \pm 11.1 \pm 5.2			
[QCDSR] [46]	246 \pm 6			
[RPM] [47]	256 \pm 26			
[QCDSR] [48]	245.3 \pm 15.7			
[LQCD] [49]	244 \pm 8			
[LQCD] [50, 51]	248.0 \pm 2.5			
[LQCD] [52]	260.1 \pm 10.8			
[LFQM] [53]	264.5 \pm 17.5			
[QCDSR] [54]	241 \pm 12			
[RBSM] [18]	248 \pm 27			

[QCDSR]- QCD sum rule.
[RPM]- Relativistic potential Model.
[LQCD]- Lattice QCD.
[LFQM]- Light front quark model.
[RBSM]- Relativistic Bethe-Salpeter Method.

VI. RESULTS AND DISCUSSION

We have studied the mass spectra and decay properties of the D_s meson in the framework of relativistic independent quark model. Our computed D_s meson spectral states are in good agreement with the reported PDG values of known states. Though there are many excited 1^- state of D_s meson known experimentally, most of them beyond 1S states are still not understood completely. And in the case of P-wave states only 1^3P_J , 1^1P_1 , and 2^1P_1 of the D_s meson are known experimentally. Our results are also compared with other theoretical model predictions.

The predicted masses of S-wave D_s meson state 2^3S_1 (2717.8 MeV) and 2^1S_0 (2633.5 MeV) are in very good agreement with experimental result of 2710_{-7}^{+12} MeV by BaBar [31] and Belle [32] Collaborations and 2638 MeV for 2^1S_0 by SELEX Collaboration [2] respectively. The

expected results of other S-wave excited states of D_s meson are also in good agreement with other reported values [13, 28–30]. The predicted P-wave D_s meson states, 1^3P_2 (2584.7 MeV), 1^3P_1 (2516.9 MeV), 1^3P_0 (2349.0 MeV) and 1^1P_1 (2435.6 MeV) are in good agreement with experimental [1] results of 2571.9 ± 0.8 MeV, 2535.12 ± 0.13 MeV, 2317.8 ± 0.6 MeV and 2459.6 ± 0.6 MeV respectively. The 2^3P_1 (2986.4 MeV) and 1^3D_3 (2931.4) are nearly 50–60 MeV off with the experimental results of 3044_{-9}^{+30} MeV [4] and 2862_{-3}^{+6} MeV [4]. However their J^P values are not yet confirmed experimentally. Though our predictions of 1S, 2S and 1P states are in agreement with the experiment. The experimental state of D_{sJ}^* (2860) is found to be a mixed states of 1^3D_3 (2931.4) and 1^3D_1 (2842.0) with a mixing probability given by $\cos^2 \theta = 0.2013$ and that for D_{sJ} (3040) is a mixed state of (2^3P_2 (3107.6) and 2^3P_0 (2764.3) with a mixing probability given by $\cos^2 \theta = 0.8030$.

In the relativistic Dirac formalism, the spin degener-

TABLE VIII. The leptonic decay width and leptonic Branching Ratio (BR) of D_s meson.

Process	$\Gamma(M \rightarrow l\bar{\nu}_l)$ (keV)		BR (keV)			
	Present	[34]	Present	[13]	[34]	Experiment [1]
$D_s \rightarrow \tau\bar{\nu}_\tau$	7.508×10^{-8}	6.090×10^{-8}	5.706×10^{-2}	4.22×10^{-2}	4.3×10^{-2}	$(5.43 \pm 0.31) \times 10^{-2}$
$D_s \rightarrow \mu\bar{\nu}_\mu$	7.648×10^{-9}	6.240×10^{-9}	5.812×10^{-3}	4.25×10^{-3}	4.41×10^{-3}	$(5.90 \pm 0.33) \times 10^{-3}$
$D_s \rightarrow e\bar{\nu}_e$	1.792×10^{-13}	. . .	1.362×10^{-7}	1.00×10^{-7}	. . .	$< 1.2 \times 10^{-4}$

acy is primarily broken therefore, to have spin average masses of the different spectral states we employ the spin averaging procedure as

$$M_{CW} = \frac{\sum_J (2J+1)M_J}{\sum_J (2J+1)} \quad (45)$$

The spin average or the center of weight masses M_{CW} are calculated from the known values of the different meson states and are compared with other model prediction [29] and those predicted by lattice QCD [LQCD] [35] in Table IV. It also help us to know the different spin dependent contributions for the observed state.

The precise experimental measurements of the masses of D_s meson states provided a real test for the choice of the hyperfine and the fine structure interactions adopted in the study of D_s meson spectroscopy. Recent study of D_s meson mass splittings in lattice QCD [LQCD] [35] using 2 ± 1 flavor configurations generated with the Clover-Wilson fermion action by the PACS-CS collaboration [35] has been used for comparison. Present results as seen in Table V are in very good agreement with the respective experimental values over the lattice results [35]. In this Table, the present results on an average, are in agreement with the available experimental value within 6% of variations, while the lattice QCD predictions [35] show 20% of variations.

The magnetic transitions (M1) can probe the internal charge structure of hadrons, and therefore they will likely play an important role in determining the hadronic structures of D_s meson. The present M1 transitions widths of D_s meson states as listed in Table VI are in accordance with the model prediction of [40] while the upper bound provided by PDG [1] is very wide. We do not find any theoretical predictions for M1 transition width of excited states for comparison. Thus we only

look forward to see future experimental support to our predictions.

The calculated pseudoscalar decay constant (f_P) of D_s meson is listed in Table (VII) along with other model predictions as well as experimental results. The value of $f_{D_s}(1S) = 252.81$ MeV obtained in our present study is in very good agreement with the experimental values provided by Belle [42], BaBar [43] and CLEO-c [44, 45]. The present value is also in accordance with other theoretical predictions for $1S$ state. The predicted f_{D_s} for higher S-wave states are found to increase with energy. However, there are no experimental or theoretical values available for comparison. Another important property of D_s meson studied in the present case is the leptonic decay widths. The present branching ratios for $D_s \rightarrow \tau\bar{\nu}_\tau$ (5.706×10^{-2}) and $D_s \rightarrow \mu\bar{\nu}_\mu$ (5.812×10^{-3}) are in excellent agreement with the experimental results ($5.43 \pm 0.31 \times 10^{-2}$ and $(5.90 \pm 0.33) \times 10^{-3}$ respectively over other theoretical predictions vide Table VIII. Large experimental uncertainty in the electron channel make it difficult for any reasonable conclusion.

Finally we look forward to see future high luminosity improved statistics and higher confidence level experimental data in support of our prediction on the spectroscopy and decay properties of the open charm-strange meson.

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